Exact Ergodic Capacity Analysis for Cognitive Underlay Amplify-and-Forward Relay Networks over Rayleigh Fading Channels

Vo Nguyen Quoc Bao and Vu Van San

Abstract: **In this paper, we propose a novel derivation approach to obtain the exact closed form expression of ergodic capacity for cognitive underlay amplify-and-forward (AF) relay networks over Rayleigh fading channels. Simulation results are performed to verify the analysis results. Numerical results are provided to compare the system performance of cognitive underlay amplify-and-forward relay networks under both cases of AF and decode-and-forward (DF) confirming that the system with DF provides better performance as compared with that with AF.**

Index Terms: **Ergodic Capacity, Rayleigh fading channels, Cognitive radio, Underlay relay networks, Amplify-and-Forward, Decode-and-Forward.**

I. Introduction

Cognitive radio has been widely considered as a promising technology for next generation mobile networks due to its superior spectral efficiency [1], [2]. There are three conventional spectrum sharing approaches including underlay, overlay, and interweave, where the underlay approach is often of interest in practical implementation. The key idea of the cognitive underlay approach is to allow unlicensed networks to transmit/receive data on the same frequency band licensed to primary (licensed) networks if its interference to the primary users is below a certain level.

An important issue in cognitive underlay networks that has attracted much attention recently is to guarantee the secondary network performance and coverage due to the constraint of the transmit powers for secondary networks, i.e., the maximum allowable interference at the primary receiver [3]. As a results, many advantaged techniques for physical layer are proposed for cognitive underlay networks, e.g., see [4], [5], [6], [7], [8], [9], [10], [11]. Although all above-mentioned research works have derived the system performance in terms of outage probability and ergodic capacity, its form for ergodic capacity is still expressed under asymptotic or upper-bound expression due to the complicated form of the end-to-end signal-to-noise ratio (SNR) of dual-hop amplify-and-forward (AF) relaying [12]. As an alternative, ergodic capacity of underlay decode-and-forward (DF) systems is usually employed to estimate that of underlay AF systems at high SNR regime leading to the fact that we cannot understand the performance gap between AF and DF.

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To the best of the authors' knowledge, exact and general ergodic capacity analysis of cognitive underlay dual-hop relaying over Rayleigh fading channels remains an open problem. This paper is to fill this important gap, i.e., providing the exact closed-form expression of the system ergodic capacity in terms of dilogarithm functions [13], [14]. All analytical results developed in this paper are corroborated by MATLAB based-simulation results, verifying the accuracy of the proposed derivation approach and the provided results. Numerical results are provided to investigate the effect of channel and system settings on the system ergodic capacity.

The main contributions of this paper are summarized as follows:

- proposing a novel approach to derive the exact closed form for system ergodic capacity over Rayleigh fading channels;
- comparing the performance of cognitive underlay dual-hop relaying networks in terms of ergodic capacity under amplify-and-forward and decode-and-forward.

It should be noted that the proposed approach not only applicable for Rayleigh fading channels but also for other generalized fading channels, i.e., Nakagami-m and Rician.

The rest of this paper is organized as follows. In Sect. II, we introduce the model under study and describe the proposed protocol. Section III shows the formulas allowing for evaluation of the system ergodic capacity over Rayleigh fading channels. In Sect. IV, we contrast the simulations and the results yielded by theory. Finally, the paper is closed in Sect. V.

II. System Model

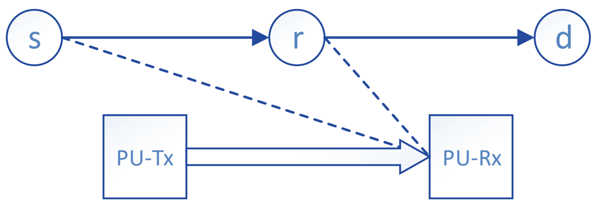

Fig. 1. Cognitive underlay AF relay network.

We consider a cognitive underlay dual hop relaying system with single amplify-and-forward relay, which is illustrated in Fig. 1. Assuming no direct link existed, the communication between the source (s) and the destination (d) is performed with the help of the relay (r) via a time-division multiple -access. In particular, the source broadcast its signal in the first time slot with transmit power $P_{\rm s}$, which is received by the relay. After amplifying with the variable gain, the scaled signal is forwarded to the the destination with transmit power $P_{\rm r}$

Denote $h_{\rm sp}$ and $h_{\rm rp}$ as the channel coefficients of the link from the source and the relay to the primary node, respectively, the transmit powers for the source and the relay under underlay mode of cognitive networks can be set as follows [15]:

$$P_{\rm s} = \frac{I_{\rm p}}{|h_{\rm sp}|^2} \tag{1}$$

and

$$P_{\rm r} = \frac{I_{\rm p}}{|h_{\rm rp}|^2}, \tag{2}$$

where $I_{\rm p}$ denotes the interference temperature constraint at the primary receiver (PU-Rx). The instantaneous signal-to-noise ratio (SNR) of the first and second hops are respectively given by

$$\gamma_1 = \frac{I_{\rm p}}{N_0}\frac{|h_{\rm sr}|^2}{|h_{\rm sp}|^2} \tag{3}$$

and

$$\gamma_2 = \frac{I_{\rm p}}{N_0}\frac{|h_{\rm rd}|^2}{|h_{\rm rp}|^2}, \tag{4}$$

where $h_{\rm sp}$ and $h_{\rm rp}$ are the channel coefficients of the link from s $\to$ s and r $\to$ d, respectively. Here, we denote N_0 as the additive Gaussian white noise power. Underlay Rayleigh fading channels, $h_{\mathcal{AB}}$ with $\mathcal{A} \in \{\rm s, r\}$ and $\mathcal{B} \in \{\rm p, r, d\}$ is a complex Gaussian random variable, so the channel power $|h_{\mathcal{AB}}|^2$ is an exponential distributed random variable with expected value $\lambda_{\mathcal{AB}} = \mathbb{E}\{|h_{\mathcal{AB}}|^2\}$ with $\mathbb{E}\{.\}$ being the expectation operator. Assuming that the path loss effect is taken into account, we have $\lambda_{\mathcal{AB}} = d_{\mathcal{AB}}^{-\eta}$, where η and $d_{\mathcal{AB}}$ denote the path-loss exponent and the distance between node $\mathcal{A}$ to node $\mathcal{B}$, respectively [16].

In dual-hop relaying networks with non-regenerative variable gain relays, the instantaneous CSI of the first hop is utilized to ensure the fixed power at the output of the relay. Accordingly, the effective SNR received at d can be expressed as [17]

$$\gamma_{\rm AF} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}. \tag{5}$$

III. Capacity Analysis

A. Amplify-and-Forward

Throughout the paper, we assume that the data channel is unknown at the transmitter but perfectly known at the receiver. Considered as the important metric in designing wireless networks, ergodic capacity reveals the upper bound on the amount of information, which can be reliably transmitted over noisy wireless channels with a certain probability of error. It is well-known that if the probability density function (PDF) of the end-to-end SNR is available, the ergodic capacity (in bits/second) per unit bandwidth can be calculated by evaluating the integral, which is of the form

$$\begin{aligned}\mathcal{C}_{\rm AF} &= \frac{1}{2}\mathbb{E}_{\gamma_{\rm AF}}\{\log(1+\gamma_{\rm AF})\}\\ &= \frac{1}{2}\int_0^\infty \log(1+\gamma) f_{\gamma_{\rm AF}}(\gamma)d\gamma,\end{aligned} \tag{6}$$

where the pre-factor 1/2 is included to reflect the fact that the source-destination communication occurs in two orthogonal time slots. However, with the current form of $\gamma_{\rm AF}$ in (5), an exact closed-form evaluation of (6) is a challenging mathematical problem due to the complexity of statistical distribution of $\gamma_{\rm AF}$ along with the presence of the nonlinear log function.

To deal with such problem, (6) should be expressed in a more mathematically tractable form. To achieve this, the basic properties of log function is employed after a some manipulations, yielding

$$\begin{aligned}\mathcal{C}_{\rm AF} =& \frac{1}{2\ln 2}\mathbb{E}_{\gamma_{\rm AF}}\left[\ln\left(1+\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2+1}\right)\right]\\ =& \frac{1}{2\ln 2}\underbrace{\mathbb{E}_{\gamma_1}\left[\ln\left(1+\gamma_1\right)\right]}_{\mathcal{C}_1}\\ &+\frac{1}{2\ln 2}\underbrace{\mathbb{E}_{\gamma_2}\left[\ln\left(1+\gamma_2\right)\right]}_{\mathcal{C}_2}\\ &-\frac{1}{2\ln 2}\underbrace{\mathbb{E}_{\gamma_1+\gamma_2}\left[\ln\left(1+\gamma_1+\gamma_2\right)\right]}_{\mathcal{C}_3}.\end{aligned} \tag{7}$$

Theorem 1: The exact closed-form expression of ergodic capacity of cognitive underlay AF networks over Rayleigh fading channels is

$$\begin{aligned}\mathcal{C}_{\rm AF} =& \frac{1}{2\ln 2}\left[\frac{\alpha_1\ln\alpha_1}{\alpha_1-1}+\frac{\alpha_2\ln\alpha_2}{\alpha_2-1}-\frac{(\alpha_1+\alpha_2)\ln(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2-1}\right]\\ &+\frac{\alpha_1\alpha_2\log(\alpha_1\alpha_2)\left[1-(\alpha_1+\alpha_2)+(\alpha_1+\alpha_2)\ln(\alpha_1+\alpha_2)\right]}{2\ln 2(\alpha_1+\alpha_2)(\alpha_1+\alpha_2-1)^2}\\ &-\frac{\alpha_1\alpha_2\left[\log(1+\alpha_1)+\log(1+\alpha_2)\right]}{2\ln 2(1-\alpha_1-\alpha_2)(\alpha_1+\alpha_2)}\\ &-\frac{\alpha_1\alpha_2\left[\mathcal{K}(\alpha_1,\alpha_2)+\mathcal{K}(\alpha_2,\alpha_1)\right]}{2\ln 2(1-\alpha_1-\alpha_2)^2},\end{aligned} \tag{8}$$

where $\mathcal{K}(\alpha_1,\alpha_2)$ is given in (9) shown at the top of the next page with ${\rm Li}_2(z) = \int_z^0 \frac{\log(1-t)dt}{t}$ denoting the dilogarithm function [18, Eq. 27.7.1].

Proof: To derive (1), we need to know the PDF of γ_1, γ_2 and $\gamma_1+\gamma_2$. For γ_1 and γ_2, recalling the results in [19], their PDF can be given by

$$f_{\gamma_i}(\gamma) = \frac{\alpha_i}{(\gamma+\alpha_i)^2}, \tag{10}$$

$$\mathcal{K}(\alpha_1,\alpha_2) = \begin{cases} -\frac{\ln^2(1-\alpha_1)}{2} + \frac{\ln^2\alpha_2}{2} + \ln\alpha_1 \ln\left[\frac{(1-\alpha_1)(\alpha_1+\alpha_2)}{\alpha_2}\right] - \mathrm{Li}_2\left(\frac{\alpha_1}{\alpha_1-1}\right) + \mathrm{Li}_2\left(-\frac{\alpha_1}{\alpha_2}\right), & \alpha_1<1 \\ \frac{\pi^2}{6} + \frac{\ln^2\alpha_2}{2} + \mathrm{Li}_2\left(-\frac{1}{\alpha_2}\right), & \alpha_1 = 1 \\ \frac{\pi^2}{2} - \frac{\ln^2(\alpha_1-1)}{2} + \frac{\ln^2\alpha_2}{2} + \ln\alpha_1\ln\left[\frac{(\alpha_1-1)(\alpha_1+\alpha_2)}{\alpha_2}\right] - \Re\left\{\mathrm{Li}_2\left(\frac{\alpha_1}{\alpha_1-1}\right)\right\} + \mathrm{Li}_2\left(-\frac{\alpha_1}{\alpha_2}\right), & \alpha_1>1 \end{cases} \quad (9)$$

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where $\alpha_1 = \frac{I_{\rm p}}{N_0}\frac{\lambda_{\rm sr}}{\lambda_{\rm sp}}$ and $\alpha_2 = \frac{I_{\rm p}}{N_0}\frac{\lambda_{\rm rd}}{\lambda_{\rm rp}}$.

For $\gamma_1+\gamma_2$, since the moment generating function (MGF) approach [20][1], could not be used due to its high derivation complexity, we start from the definition of the cumulative distribution function (CDF) of $\gamma_1+\gamma_2$ and invoke the concept of conditional probability, namely

$$F_{\gamma_1+\gamma_2}(\gamma) = \Pr(\gamma_1+\gamma_2<\gamma) = \int_0^{\gamma} F_{\gamma_1}(\gamma - x) f_{\gamma_2}(x)dx. \quad (11)$$

In (11), $F_{\gamma_1}(.)$ is the CDF of γ_i, which is obtained by integrating (10) from 0 to γ as follows:

$$F_{\gamma_1}(\gamma) = \frac{\gamma}{\gamma+\alpha_1}. \quad (12)$$

Substituting (12) and (10) into (11), after some manipulations, we have

$$F_{\gamma_1+\gamma_2}(\gamma) = \frac{\gamma}{\gamma+\alpha_1+\alpha_2} + \frac{\alpha_1\alpha_2}{(\gamma+\alpha_1+\alpha_2)^2}\left[\log\frac{\alpha_1}{\gamma+\alpha_1} + \log\frac{\alpha_2}{\gamma+\alpha_2}\right]. \quad (13)$$

Employing the relationship between the PDF and the CDF, we can obtain the PDF of $\gamma_1+\gamma_2$ as

$$f_{\gamma_1+\gamma_2}(\gamma) = \frac{\alpha_1+\alpha_2}{(\gamma+\alpha_1+\alpha_2)^2} - \frac{2\alpha_1\alpha_2\log(\alpha_1\alpha_2)}{(\gamma+\alpha_1+\alpha_2)^3} - \frac{\alpha_1\alpha_2}{(\gamma+\alpha_1+\alpha_2)^2}\left(\frac{1}{\gamma+\alpha_1}+\frac{1}{\gamma+\alpha_2}\right) + \frac{2\alpha_1\alpha_2}{(\gamma+\alpha_1+\alpha_2)^3}\left[\log(\gamma+\alpha_1)+\log(\gamma+\alpha_2)\right]. \quad (14)$$

Having the PDFs of γ_1, γ_2 and $\gamma_1+\gamma_2$ in hands allows us to derive $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$. We easily recognize that $\mathcal{C}_1$ and $\mathcal{C}_2$ take the general form, which is written as

$$\mathcal{J}(a) = \int_0^\infty \ln(1+\gamma)\frac{a}{(\gamma+a)^2}d\gamma. \quad (15)$$

With the help of the identity [18, Eq. (4.291.17)], we have

$$\mathcal{J}(a) = \begin{cases} \frac{a\ln a}{a-1}, & a\neq 1 \\ 1, & a = 1 \end{cases}. \quad (16)$$

[1]The PDF can be yielded by taking the inverse Laplace transform of the MGF of $\gamma_1+\gamma_2$, which can be obtained as the product of the MGF of its summands.

We are now in a position to derive $\mathcal{C}_3$. Rewriting $\mathcal{C}_3$ in an explicit form, we obtain

$$\begin{aligned}\mathcal{C}_3 = &\int_0^\infty \ln(1+\gamma) f_{\gamma_1+\gamma_2}(\gamma)d\gamma \\ =&(\alpha_1+\alpha_2)\underbrace{\int_0^\infty \frac{\ln(1+\gamma)}{(\gamma+\alpha_1+\alpha_2)^2}d\gamma}_{\mathcal{I}_1} \\ &- 2\alpha_1\alpha_2\log(\alpha_1\alpha_2)\underbrace{\int_0^\infty\frac{\ln(1+\gamma)}{(\gamma+\alpha_1+\alpha_2)^3}d\gamma}_{\mathcal{I}_2} \\ &- \alpha_1\alpha_2\int_0^\infty \frac{\ln(1+\gamma)}{(\gamma+\alpha_1+\alpha_2)^2}\left(\frac{1}{\gamma+\alpha_1}+\frac{1}{\gamma+\alpha_2}\right)d\gamma \\ &+ 2\alpha_1\alpha_2\underbrace{\int_0^\infty \frac{\ln(1+\gamma)\left[\log(\gamma+\alpha_1)+\log(\gamma+\alpha_2)\right]}{(\gamma+\alpha_1+\alpha_2)^3}d\gamma}_{\mathcal{I}_3}.\end{aligned} \quad (17)$$

where $\mathcal{I}_i$ with $i = 1,2,3$ are auxiliary functions, which are derived next.

Starting with $\mathcal{I}_1$ and using the identity [18, Eq. (4.291.17)], we have

$$\mathcal{I}_1 = \begin{cases} \frac{\ln(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2-1}, & \alpha_1+\alpha_2\neq 1 \\ 1, & \alpha_1+\alpha_2 = 1 \end{cases}. \quad (18)$$

To solve $\mathcal{I}_2$, based on integration by parts, we obtain

$$\begin{aligned}\mathcal{I}_2 =& -\left.\frac{\ln(1+\gamma)}{2(\gamma+\alpha_1+\alpha_2)^2}\right|_{\gamma=0}^{\infty} + \frac{1}{2}\int_0^\infty \frac{d\gamma}{(\gamma+1)(\gamma+\alpha_1+\alpha_2)^2} \\ =& \begin{cases} \frac{1-(\alpha_1+\alpha_2)+(\alpha_1+\alpha_2)\ln(\alpha_1+\alpha_2)}{2(\alpha_1+\alpha_2)(\alpha_1+\alpha_2-1)^2}, & \alpha_1+\alpha_2\neq 1 \\ \frac{1}{4}, & \alpha_1+\alpha_2 = 1\end{cases}.\end{aligned} \quad (19)$$

Using integration by parts again, $\mathcal{I}_3$ is re-expressed as

$$\mathcal{I}_3 = \underbrace{-\frac{\ln(1+\gamma)\left[\log(\gamma+\alpha_1)+\log(\gamma+\alpha_2)\right]}{2(\gamma+\alpha_1+\alpha_2)^2}\Bigg|_{\gamma=0}^{\infty}}_{\to 0} + \frac{1}{2}\int_0^\infty \frac{\ln(1+\gamma)}{(\gamma+\alpha_1+\alpha_2)^2}\left[\frac{1}{\gamma+\alpha_1}+\frac{1}{\gamma+\alpha_2}\right]d\gamma + \frac{1}{2}\underbrace{\int_0^\infty \frac{\log(\gamma+\alpha_1)+\log(\gamma+\alpha_2)}{(1+\gamma)(\gamma+\alpha_1+\alpha_2)^2}d\gamma}_{\mathcal{I}_4}. \tag{20}$$

Plugging (20) into (17) and then canceling the like terms, $\mathcal{C}_3$ is simplified as

$$\mathcal{C}_3 = (\alpha_1+\alpha_2)\mathcal{I}_1 - 2\alpha_1\alpha_2\log(\alpha_1\alpha_2)\mathcal{I}_2 + \alpha_1\alpha_2\mathcal{I}_4. \tag{21}$$

For $\mathcal{I}_4$, by employing partial fraction decomposition, we have

$$\mathcal{J}_4 = \frac{1}{1-\alpha_1-\alpha_2}\int_0^\infty \frac{\log(\gamma+\alpha_1)+\log(\gamma+\alpha_2)}{(\gamma+\alpha_1+\alpha_2)^2}d\gamma + \frac{1}{(1-\alpha_1-\alpha_2)^2}\underbrace{\int_0^\infty\left(\frac{\log(\gamma+\alpha_1)}{\gamma+1}-\frac{\log(\gamma+\alpha_1)}{\gamma+\alpha_1+\alpha_2}\right)d\gamma}_{\mathcal{K}(\alpha_1,\alpha_2)} + \frac{1}{(1-\alpha_1-\alpha_2)^2}\underbrace{\int_0^\infty\left(\frac{\log(\gamma+\alpha_2)}{\gamma+1}-\frac{\log(\gamma+\alpha_2)}{\gamma+\alpha_1+\alpha_2}\right)d\gamma}_{\mathcal{K}(\alpha_2,\alpha_1)}. \tag{22}$$

In (22), we can see that the second and third integral take from the general form, given as follows:

$$\mathcal{K}(a,b) = \int_0^\infty\left(\frac{\log(\gamma+a)}{\gamma+1}-\frac{\log(\gamma+a)}{\gamma+a+b}\right)d\gamma. \tag{23}$$

By recognizing the integral representation of the dilogarithm function [21, Eq. (27.7.1)], i.e., $\mathrm{Li}_2(-x) = \int_1^x \frac{\ln t dt}{t-1}$, after some manipulation, we find out that $\mathcal{K}(a,b)$ can be derived as shown in (24) at the top of this page. Furthermore, making use the fact that $\mathrm{Im}\{\mathrm{Li}_2[a/(a-1)]\} = 2\pi \mathrm{arc}\cot(1-2a)$ for $a>1$, (24) can be rewritten as (9), where $\Re(.)$ and $\Im(.)$ represent the real and image part of a complex number, respectively.

For the case of $\alpha_1+\alpha_2=1$, it is straightforward to arrive

$$\mathcal{K}(a,b) = \int_0^\infty \frac{\log(\gamma+\alpha_1)+\log(\gamma+\alpha_2)}{(1+\gamma)^3}d\gamma = \frac{\alpha_1-1+(\alpha_1-2)\alpha_1\log\alpha_1}{2(\alpha_1-1)^2} + \frac{\alpha_2-1+(\alpha_2-2)\alpha_2\log\alpha_2}{2(\alpha_2-1)^2}. \tag{25}$$

□

Pulling everything together, i.e., (6), (16), and (17), we can obtain the exact closed-form expression for $\mathcal{C}_{\mathrm{AF}}$. It should be noted that the dilogarith function is available as a build-in function in most well-known mathematical softwares such as MATLAB and MATHEMATICA. Beside, there exists efficient approaches to directly calculate the dilogarith, e.g., see [13], [14].

B. Amplify-and-Forward versus Decode-and-Forward

In this section, we provide the ergodic capacity for dual hop DF networks, which are considered as a counterpart of AF networks. The ergodic capacity of the underlay DF network is defined as

$$\mathcal{C}_{\mathrm{DF}} = \frac{1}{2}\int_0^\infty \log_2(1+\gamma) f_{\gamma_{\mathrm{DF}}}(\gamma)d\gamma, \tag{26}$$

where γ_{DF} is the equivalent end-to-end SNR of the system.

It is well-known that the exact form of γ_{DF} in terms of γ_1 and γ_2 is not mathematically visible. To proceed further, we adopt the mathematical tractability approximation approach suggested by Wang *et. al.* [22]. In particular, the equivalent end-to-end SNR of re-generative relaying systems, γ_{DF}, can be tightly approximated irrespective of the employed modulation scheme as

$$\gamma_{\mathrm{DF}} = \min(\gamma_1, \gamma_2). \tag{27}$$

The PDF of γ_{DF} is given by

$$f_{\gamma_{\mathrm{DF}}}(\gamma) = \frac{d}{d\gamma}\left[1-(1-F_{\gamma_1}(\gamma))(1-F_{\gamma_2}(\gamma))\right] = \frac{\alpha_1\alpha_2}{(\gamma+\alpha_1)(\gamma+\alpha_2)^2} + \frac{\alpha_1\alpha_2}{(\gamma+\alpha_2)(\gamma+\alpha_1)^2} \overset{(a)}{=} \frac{\alpha_1\alpha_2}{\alpha_2-\alpha_1}\left[\frac{1}{(\gamma+\alpha_1)^2}-\frac{1}{(\gamma+\alpha_2)^2}\right], \tag{28}$$

where (a) immediately follows after using partial fraction technique with some simple manipulations.

Theorem 2: The ergodic capacity of cognitive underlay DF relay networks is tightly approximated by

$$\mathcal{C}_{\mathrm{DF}} = \frac{\alpha_1\alpha_2}{2\ln 2(\alpha_2-\alpha_1)}\left(\frac{\log\alpha_1}{\alpha_1-1}-\frac{\log\alpha_2}{\alpha_2-1}\right). \tag{29}$$

Proof: Plugging (28) into (26) and taking the integral, we have (after simplifications).

For $\alpha_1=\alpha_2=\alpha$, $\mathcal{C}_{\mathrm{DF}}$ is simplified as

$$\mathcal{C}_{\mathrm{DF}} = \begin{cases} \frac{\alpha(1-\alpha+\alpha\log\alpha)}{2\ln 2(\alpha-1)^2}, & \alpha\neq 1 \\ \frac{1}{2\ln 2}, & \alpha = 1 \end{cases}. \tag{30}$$

□

Having $\mathcal{C}_{\mathrm{AF}}$ and $\mathcal{C}_{\mathrm{DF}}$ in hands allows us to numerically evaluate the system ergodic capacity for a given network and channel condition,

$$\mathcal{K}(a,b) = \begin{cases} -\frac{\ln^2(1-a)}{2} + \frac{\ln^2 b}{2} + \ln a \ln\left[\frac{(a-1)(a+b)}{b}\right] - \mathrm{Li}_2\left(\frac{a}{a-1}\right) + \mathrm{Li}_2\left(-\frac{a}{b}\right), & a<1 \\ \frac{\pi^2}{6} + \frac{\ln^2 b}{2} + \mathrm{Li}_2\left(-\frac{1}{b}\right), & a=1 \\ \frac{\pi^2}{2} + 2\mathrm{i}\pi \operatorname{arc\,cot}(1-2a) - \ln^2(a-1) + \ln^2 b - 2\ln a \ln\left[\frac{b}{(a-1)(a+b)}\right] - \mathrm{Li}_2\left(\frac{a}{a-1}\right) + \mathrm{Li}_2\left(-\frac{a}{b}\right), & a>1 \end{cases} \tag{24}$$

IV. Numerical Results and Discussion

This section is to verify the proposed derivation approach and to study effects of the system and channel settings on the system ergodic capacity. We consider the 2-D system model, where the source, the relay, the destinations and the primary receiver are placed at coordinate (0,0), $(d,0)$, (1,0), and $(x_{\rm p}, y_{\rm p})$, respectively. Recalling that here we adopt the simplified path loss model, we can model $\lambda_{\mathcal{AB}} = d_{\mathcal{AB}}^{-\eta}$, where $d_{\mathcal{AB}}$ denotes the distance between node $\mathcal{A}$ to node $\mathcal{B}$ and η is the path-loss exponent.

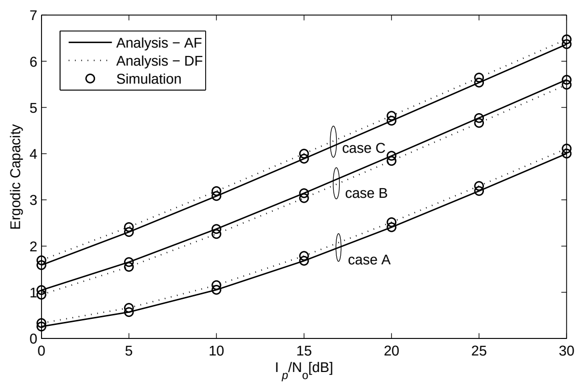

Fig. 2. Ergodic capacity versus average SNRs.

In Fig. 2, we plot the system ergodic capacity versus average SNRs in dB for three different cases of the primary user location, i.e., case A: (0.3, 0.3), case B: (0.6, 0.6), and case C: (0.9, 0.9). It can be seen that that the simulation results are in excellent argument with the analysis results for both cases of AF and DF confirming the correctness of the proposed derivation approach. We also see that case A outperforms case B, which, in turns, outperforms case C suggesting that the system capacity can improves when the secondary networks are located far away from the primary receivers, as expected. For the same channels and system settings, the system using AF provides better capacity as compared with that using DF. It can be explained by using the fact that DF can eliminate noise when make right decoding, while AF amplifies the noise when forwarding.

In Fig. 3, we investigate the effect of secondary relay position the system ergodic capacity for a fixed coordinate of the primary receiver. We can see that the best location of secondary relays is a complicated function of $I_{\rm p}$ and relaying technique used.

In Fig. 4, we investigate the effect of the location of the primary receiver on the system performance. We consider

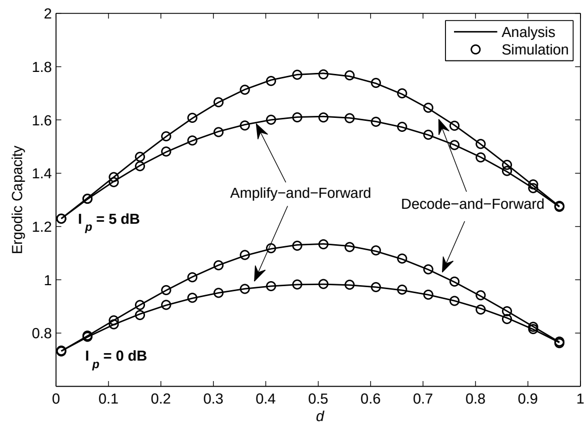

Fig. 3. Effect of relay locations on the system ergodic capacity.

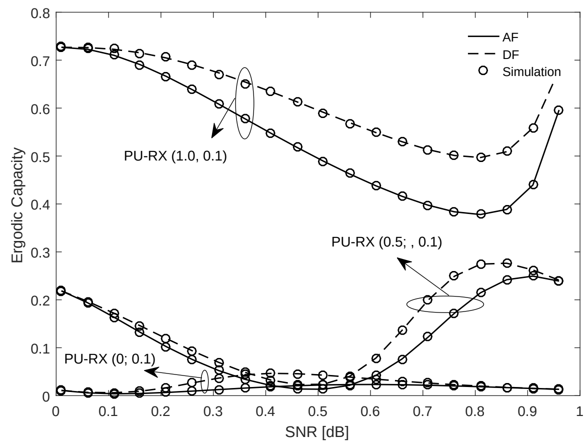

Fig. 4. Effect of relay locations on the system ergodic capacity.

three extreme cases for the primary receiver, i.e., very close to the secondary source, very close to the secondary relay, very close to the relay, and very close to the secondary destination. Among three cases, the last case provides the best performance while the first case gives the worst performance. It can be explained by making use the fact that the primary receiver locations in relation to the secondary transmitter significantly affect the secondary system performance.

V. Conclusion

This paper has proposed a novel derivation approach to study the performance of cognitive underlay amplify-and-forward relay networks. In addition, our approached approach is applicable for other fading channels models. From the obtained results, we can conclude that DF should be used at high SNR regime to provide high system capacity as compared with AF.

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